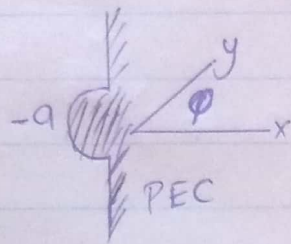
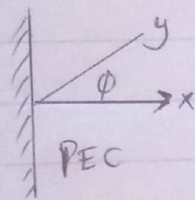


5. (20) An infinite PEC plane resides in the $y-z$ plane; while most of the structure is planar, a semi-cylindrical protrusion of radius a exists along the z -axis. A plane wave polarized along the z -axis is incident normally upon the PEC plane.

$$E_z^{io} = E_0 e^{-jkx}$$



(a) (5) Determine the total incident electric field E_z^i without the semi-cylindrical protrusion, but with half-space PEC.
 Soln: Without the semi-cylindrical protrusion;



The incident wave lies in the $y-x$ plane

From the electric field formula;

$$(i) \dots E_z^i(r) = E_0(r_0) \left(\frac{\rho_1^i \rho_2^i}{(\rho_1^i + x_i)(\rho_2^i + x_i)} \right)^{1/2} \cdot e^{-jk(s(r_0) + x_i)}$$

Since it's a planar wave in a PEC, both electrical conductivities are infinite. Therefore;

$$(ii) \dots \rho_1^i = \rho_2^i = \infty$$

Equation (i) is reduced to:

$$E_z^i(r) = E_0(r_0) \left(\frac{\infty \cdot \infty}{(\infty + x_i)(\infty + x_i)} \right)^{1/2} \cdot e^{-jkx_i}$$

$$\text{Hence, } E_z^i = E_0 \cdot (1)^{1/2} \cdot e^{-jkx_i} = \underline{\underline{E_0 e^{-jkx_i}}}$$

(b) (B) Transform the field in (a) in the form of standing cylindrical waves and simplify it.

Solution

$$\text{From (a)} : E_z^{i0} = E_0 e^{-jkx}$$

In cylindrical form, $x = \rho \cos \phi$

$$\text{Hence : } E_z^{i0} \Rightarrow E_0 e^{-jk\rho \cos \phi}$$

$$E_z^{i0} = E_0 \sum_{n=-\infty}^{\infty} a_n J_n(B\rho) e^{jn\phi}$$

N/B: It's a Perfect Electrical Conductor (PEC) hence the electrical conductivities are infinite.

$$E_i = \hat{a}_z \cdot E_z^{i0}$$

$$E_i = \hat{a}_z \cdot E_0 \sum_{n=-\infty}^{\infty} a_n J_n(B\rho) e^{jn\phi}$$

Simplifying the above equation :

$$E_i = \hat{a}_z E_0 \sum_{n=0}^{\infty} (-j)^n \epsilon_n J_n(B\rho) \cos(n\phi)$$

$$\text{Where : } \epsilon_n = \begin{cases} 1 & n=0 \\ 2 & n \neq 0 \end{cases}$$

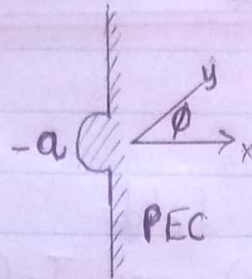
$$H_i = -\frac{1}{j\omega\mu} \nabla \times E_i$$

Therefore ;

$$H_i = -\frac{\beta E_0}{j\omega\mu} \sum_{n=0}^{\infty} j^{-n} J'_n(B\rho) e^{jn\phi}$$

(c) (5) Give the form of the scattered field due to the protrusion and the boundary conditions to decide the unknown coefficients

Solution



Radius = a

Total Field, $E^t = E^i + E^s$

where E^i = Incident electric field

E^s = Scattered electric field

$$E_z^s = \hat{a}_z \cdot E_z^s = \hat{a}_z \cdot E_0 \sum_{n=-\infty}^{+\infty} a_n J_n(\beta \rho)$$

Applying boundary conditions;

$$E^t = \hat{a}_z \cdot E_z^t (\rho = a, 0 \leq \phi \leq 2\pi, z) = 0$$

$$= E_0 \sum_{n=-\infty}^{\infty} [j^{-n} J_n(\beta a) e^{jn\phi} + C_n H_n^{(2)}(\beta a)] = 0$$

Scattered electric field, E_z^s reduces to :

$$E_z^s = -E_0 \sum_{n=-\infty}^{\infty} (-j)^n \epsilon_n \frac{J_n(\beta a)}{H_n^{(2)}(\beta a)} H_n^{(2)}(\beta \rho) \cos(n\phi)$$

$$\text{where } \epsilon_n = \begin{cases} 1 & n = 0 \\ 2 & n \neq 0 \end{cases}$$

Scattered magnetic field :

$$H_\phi^s = \frac{1}{j\omega\mu} \cdot \frac{\partial E_z^s}{\partial \rho}$$

Therefore :

$$H_{\phi}^s = \frac{-\beta E_0}{j\omega\mu} \sum_{n=-\infty}^{\infty} j^{-n} \frac{J_n(\beta a)}{H_n^{(2)}(\beta a)} H_n^{(2)'}(\beta \rho) e^{jn\phi}$$

(d) (5) Find the scattered fields
 $[J_{-n}(x) = (-1)^n J_n(x) \text{ and } J_n(-x) = (-1)^n J_n(x)]$

Solution

Replace the values in the electric and magnetic scattered fields:

$$E_z^s = -E_0 \sum_{n=-\infty}^{\infty} (-j)^n \epsilon_n \frac{(-1) \beta a}{H_n^{(2)}(\beta a)} H_n^{(2)}(\beta \rho) \cos(n\phi)$$

Therefore :

$$E_z^s = E_0 \sum_{n=-\infty}^{\infty} (-j)^n \epsilon_n \frac{\beta \rho \cos(n\phi)}{H_n^{(2)}}$$

$$H_{\phi}^s = \frac{-\beta E_0}{j\omega\mu} \sum_{n=-\infty}^{\infty} j^{-n} \frac{J_n(\beta a)}{H_n^{(2)}(\beta a)} H_n^{(2)'}(\beta \rho) e^{jn\phi}$$

Therefore :

$$H_{\phi}^s = \frac{\beta E_0}{j\omega\mu} \sum_{n=-\infty}^{\infty} j^{-n} \frac{\beta \rho \cdot e^{jn\phi}}{H_n^{(4)}}$$